Title: Authentic Variations

Link to Outcomes:

- **Problem Solving** Students will analyze data cooperatively to develop a working model of a function.
- **Communication** Students will express verbally and in writing their predictions, comparisons, and conclusions based on the collected data.
- **Reasoning** Students will make predictions and draw conclusions using gathered data.
- **Connections** Students will connect real-life experiences to a mathematical model.
- **Estimation** Students will demonstrate their ability to apply estimation strategies in determining the appropriate model for each activity.
- **Measurement** Students will utilize a variety of measurement skills in the data collecting activities.
- **Statistics** Students will demonstrate their ability to collect, organize, display data, and to interpret information obtained from a graph.
- **Algebra** Students will demonstrate their ability to perform algebraic operations and create a model.
- **Arithmetic Operations** Students will compute predicted data using their model.
- **Patterns/ Relationships** Students will demonstrate their ability to recognize a function, represent it on a graph, and describe the relationship between two variables.
- **Technology** Students will demonstrate their ability to make a graph and determine a model on a graphing calculator.

Brief Overview:

Middle school students are often interested in the application of algebra to real-life situations. In order to show this connection, students will cooperatively participate in authentic data gathering activities. They will graph their data using a graphing calculator and determine whether the two quantities measured in a particular experiment are directly or inversely proportional. After developing working equations for the graphs, they will make predictions, find applications, and draw conclusions based on the data.

Grade/Level:

Grade 8 and advanced grade 7.

Duration/Length

5 or more class periods

Prerequisite Knowledge:

Students should have a working knowledge of the following skills:

- collecting data
- organizing data
- constructing a scatter plot
- using a graphing calculator
- using U.S. and metric measurements

Objectives:

The students will:

- work cooperatively in groups to compute data from real-life problem situations.
- display data from experiments.
- construct scatter plots on the graphing calculator.
- determine the difference between direct and inverse proportions
- formulate a linear or inverse linear mathematical model
- analyze data through predictions , comparisons, and applications

Materials/Resources/Printed Materials:

- stop watch
- worksheets: "Wave Experiment" , Labs 1-4, "Inverse Variation", "Variation Assessment"
- graphing calculators
- 10-20 lids to containers of various sizes
- 10 metric tape measures
- 6 yard sticks or customary measuring tape
- 2 sets of 2 foot tape, marked "jump line"
- index cards, marked: 4 ft, 10 ft, 15 ft, 23 ft, 27 ft, and 36 ft
- whistle or noise maker
- 2 sets of signs labeled: Lab 1, Lab 2, Lab 3, and Lab 4
- 2 bouncing balls
- 1 ruler per student or group
- 1 triangular prism per student or group
- 12 pennies per student or group
- graph paper

Development/Procedures:

Days One and Two: Introduction / Modeling Activities

- Discuss "the wave" done by fans at Oriole Park at Camden Yards. Ask what it is, when it is done, why it is done, and how it is done. (Motivation)

- Distribute and follow the "Wave Experiment" worksheet. Be careful to review prerequisite skills and explain the content completely.

- Use the "TI-80 Graphing Calculator" worksheet, when you reach direction 6, to aid the students in plotting the data, drawing the line of best fit, and finding an equation to describe the data.

- Complete the lesson by having students use the cooperative strategy, "think, pair, share" to answer the questions. [Students think about and write individual answers for the first 10 minutes. Then they pair and exchange their results with a partner or group for 10 minutes. Finally, students will share their results with the class, as the teacher leads students to expand on their ideas.] The "think" part of this activity may be completed for homework.

- Generate a summary of what was learned through questioning techniques. Tell the students that they will be working in groups tomorrow to complete these types of activities.

Days Three and Four: Student Lab Experiments

- Set up two sets of the four stations as described below :

  Lab 1 - "Lid Experiment" worksheets
  5 - 10 various lids to containers
  4 graphing calculators
  measuring tape

  Lab 2 - "Long Jump Experiment" worksheets
  A standing long jump set up, in which a 2 ft piece of tape is marked "jump line" and yard sticks are lined up in front to measure distances jumped. (A measuring tape may be used instead to measure the distances jumped.) Additionally, index cards marked " 4 ft"," 10 ft", "15 ft"," 23 ft"," 27 ft" and "36 ft". These cards should be taped to the floor behind the jump line at the appropriate running distances.
  4 graphing calculators

  Lab 3 - "Real Estate Experiment" worksheets
  4 graphing calculators
  graph paper
  Lab 4 - "Bouncing Ball Experiment" worksheets
  A metric tape measure taped vertically to the wall.
  A bouncing ball.
  4 graphing calculators.

- Arrange students in groups of 4.

- Tell students to move from lab to lab finished or not, every 20 minutes. Indicate this time by blowing a whistle or making a specified noise. Have students move in numerical order. The teacher should move around to each group, offering help if needed.

- Have each group complete 1 worksheet cooperatively. If students do not finish questions, they may finish them for homework. The beginning of the next class may be used for groups to get together to discuss and expand upon their ideas.

Day Five: Lesson Expansion / Individual Project

- Discuss results of previous days' experiments. Compare the graphs and determine which data illustrates a direct proportion.

- Take a close look at the Real Estate data. Discuss the differences in the graph; the fact that it is not a direct proportion and why; what the shape of the graph is; and, how we can determine the equation for the graph.

- Have students cooperatively complete the "Inverse Variation" worksheet.

Evaluation:

Students will be evaluated :

- by their successful completion of the experiments.

- through the use of the "Variation Assessment" worksheet.

Extension/Follow Up:

Students may develop their own experiments, collect data, graph it, determine if they are direct or inverse variations, and justify their decisions. Even though some of their choices may be neither direct or inverse, students will show their knowledge of variations by being able to explain why the situation may be neither.

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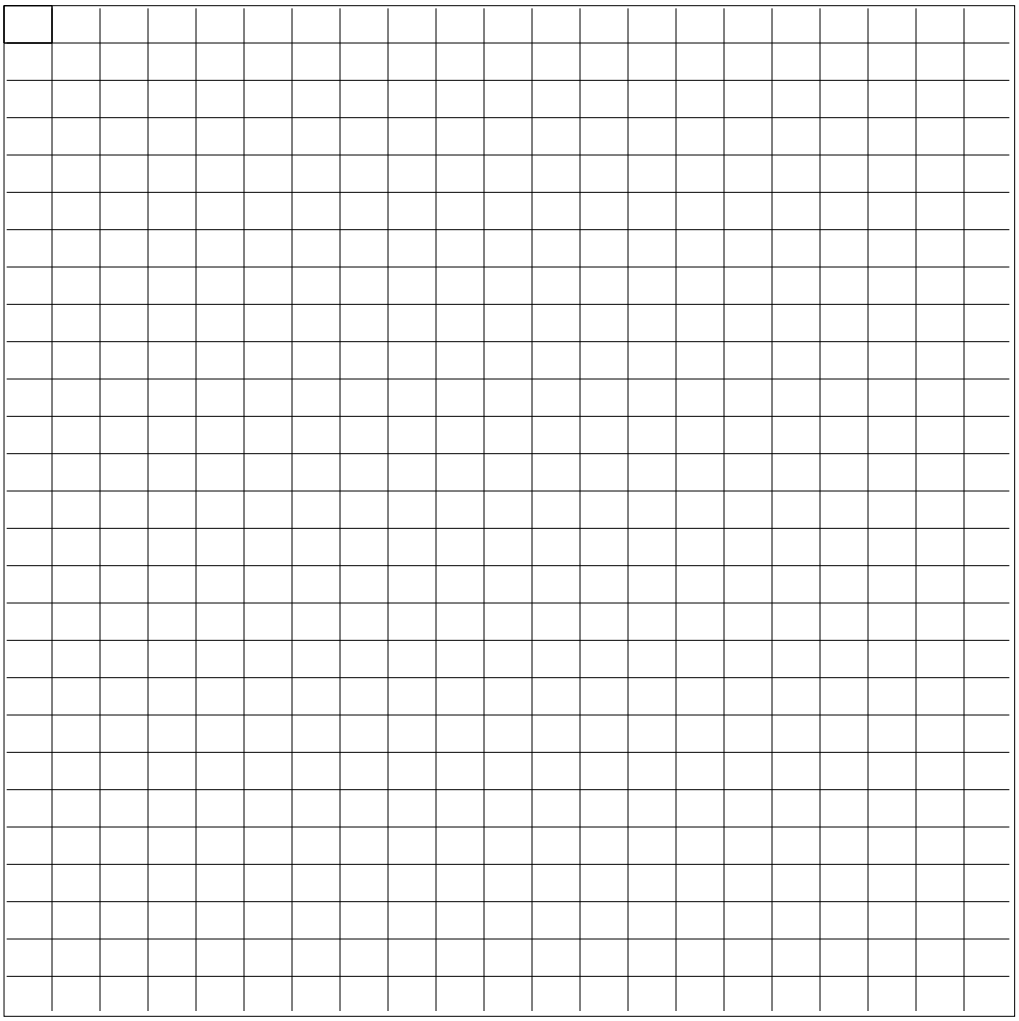
WAVE EXPERIMENT

DIRECTIONS:

1. **Arrange students in logical sections to make the "wave." The "wave" refers to the up and down movement of fans to show enthusiasm at Oriole baseball games.**
2. **The setting of the stop watch and the teacher saying "go" means for the students to proceed. The first section of students make the wave, with the last student saying "stop," as he/she sits. The stop watch is stopped on the elapsed time.**
3. **The activity should be completed with more students being added on at irregular intervals. You may want to have 4 , then 7, then 10, then 12 , etc. students making the wave. Continue adding students until the entire class has been in at least one wave.**

Number in group	Elapsed Time (seconds)

4. Plot the data on the graph using a scatter plot.
Let x be the number of people in the wave and let y be the elapsed time.



5. Draw a line of best fit.

6. Plot the data on your graphic calculator in the same manner as above. Be sure to include your line of best fit. Use the directions on the "TI-80 Graphing Calculator" worksheet.

QUESTIONS FOR WAVE EXPERIMENT:

1. **How does your line of best fit compare to the line of best fit on the calculator?**

2. **Write a sentence that describes the relationship between the number of people and the elapsed time.**

3. **Using the graphing calculator, determine an equation to describe your data.**

4. **How long would it take if 40 people participated in the wave? 100 people? 1000 people?**

5. **There is an average of 40,000 people in attendance during a game at Oriole Park at Camden Yards. If the wave were to move row by row and through each section individually, how long would it take to complete the wave?**

6. **Would the results be different at a ball park than in the classroom? Why / why not?**

7. **If the wave were to last 1 minute, estimate how many people would have participated. Estimate the number of people in 1 hour.**

8. **Of what value is there to computing elapsed time to the movement of people? Are there any occupations in which this information might be relevant?**

LID EXPERIMENT

DIRECTIONS:

1. **Find the diameter and circumference of the various lids. Record your results in the table below. Express your answer in decimal form, to the nearest mm.**

Diameter (cm)	Circumference (cm)

2. **Plot your data on your graphic calculator. Let x be the diameter and y be the circumference.**

3. **Draw the line of best fit.**

QUESTIONS FOR LID EXPERIMENT:

1. **Write a sentence that describes the relationship between the diameter and the circumference.**

2. **What is the equation for the line of best fit?**

3. **Is this line a good indication of how this data relates? Why / why not?**

4. **If this is a good indicator of how the data relates, then predict what the diameter would be if the circumference was 75 cm?**

5. **The diameter of a quarter is 2.3 cm. Using your equation, determine the circumference of the mold used to mint this coin.**

6. **Of what value is there to comparing diameter to the circumference? Are there any manufactures to which this information might be relevant?**

LONG JUMP EXPERIMENT

DIRECTIONS:

1. **Within your group decide who will take the following roles:**
   jumper- does the standing and running jump;
   measurer- measures the distance jumped;
   recorder- verifies the jump measurement and records data in the table (all measurements should be converted to feet);
   leader/judge- organizes group, starts the jumper and decides if the jump is valid. This job could be eliminated if group only has 3 members.
2. **The jumper stands on the jump line facing the tape measure.**
3. **The leader decides when the jumper goes. The measurer measures the jump and the recorder completes the table.**
4. **Next the jumper stands at the 4 ft position and makes a running jump from the jump line. The judge should make sure the jumper's foot does not go past the jump line.**
5. **The jumper continues to make running jumps from each of the positions while the other group members continue their jobs.**

Starting distance	Distance jumped (ft)
0 ft	
4 ft	
10 ft	
15 ft	
23 ft	
27 ft	
36 ft	

6. **Graph the jump measurements on the graphing calculator using a scatter plot. Let x be the starting distance and y be your jumping distance.**
7. **Draw the line of best fit.**

QUESTIONS FOR THE LONG JUMP EXPERIMENT:

1. **Write a sentence that describes the relationship between the distance from the starting point and the distance jumped.**

2. **What is the equation for the line of best fit?**

3. **Is this line a good indication of how this data relates? Why/why not?**

4. **If the line is a good indicator of how the data relates, then predict what the length of your jump would be if your starting distance from the jump line was 25 ft? 100 ft?**

   Are the length of these jumps reasonable? Why / why not?

5. **If you wanted to break the 30 ft. running long jump record made by Mike Connelly, how far from the jump line would you want to start?**

REAL ESTATE EXPERIMENT

DIRECTIONS:

1. **You are purchasing a piece of land. The real estate agent tells you the area of the land is 1000 square feet. You forgot to ask the agent for the dimensions of the property. Sketch at least 6 possible rectangular shapes the property could be with the dimensions and record the data in the chart.**

Length (cm)	Width (cm)

2. **Plot your data on your graphic calculator using a scatter plot. Let x be the length and y be the width.**

3. **Draw the line of best fit.**

QUESTIONS:

1. **Write a sentence that describes the relationship between the height and length of the property size.**

2. **What is the equation of the line of best fit?**

3. **Is this line a good indicator of how the data relates? Why or why not?**

4. **If this is a good indicator of how the data relates, then predict what the width would be if the length was 45 feet.**

BOUNCING BALL EXPERIMENT

Directions:

1. **In your group, decide who will drop the ball, who will measure the bounce, and who will record the measurement.**
2. **Hold the ball at the height of the 30 cm mark on the tape measure. Now drop the ball and measure how high it rebounds after one bounce.**
3. **Record the height on the table below.**
4. **Continue steps 2 and 3 until all the measurements are made and recorded in the table.**

Height of drop	Height after one bounce
30 cm	
45 cm	
57 cm	
75 cm	
93 cm	
105 cm	
112 cm	
126 cm	
141 cm	
150 cm	

5. **Graph the measurements on the graphing calculator using a scatter plot. Let x be the height of the drop and y be the height after one bounce.**
6. **Draw the line of best fit.**

QUESTIONS FOR BOUNCING BALL EXPERIMENT:

1. **Write a sentence that describes the relationship between the height of the drop and the height after one bounce.**

2. **What is the equation for the line of best fit?**

3. **Is this line a good indication of how this data relates? Why/ why not?**

4. **If the line is a good indicator of how the data relates, then predict what the height of the ball would be after one bounce, if you drop the ball from 300 cm? 1000 cm?**

   Are these numbers reasonable? Why/ why not?

5. **If you could be suspended from the ceiling of the Arena, how high would you want to be in order to drop a basketball and have it rebound into the basket on one bounce. Remember that a typical basketball net is 10 feet from the floor.**

INVERSE VARIATION WORKSHEET

1. **Using a graphing calculator graph and compare the following equations.**

   y=10/x

   y=25/x

   y=100/x

2. **How do these graphs compare to the graphed data from the Real Estate Experiment?**

3. **Graph the Real Estate data on a scatter plot on the calculator.**

4. **Using trial and error determine the equation for the graph. What is it?**

5. **Using this equation now determine what the width of the property would be if the length was 45 feet.**

6. **Describe in a sentence or two the relationship between data in an inverse variation.**

VARIATIONS ASSESSMENT

Directions: For each of the following activities , graph collected data on a scatter plot, determine the equation that represents the data and answer the questions.

1.

Vacation Condominium Rental (for 1 week)

Number of people staying	Cost per person
2	350
10	70
20	35
7	100
50	14

Equation?

What type of variation exists with this data? Explain how you know.

If you and five of your friends shared this condominium for a week how much would you charge each of your friends?

Explain how you came up with this amount.

2.

Investments

Amount invested (in dollars)	Interest earned (in dollars)
750	48.75
545	35.43
675	56.88
900	58.50
1500	97.50

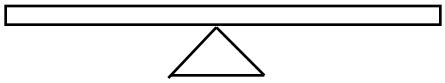
Equation?

What type of variation exists with this data?
Explain how you know.

If you had $2560 to invest how much interest would you earn?

Explain how you came up with this amount.

3. Using a ruler and a triangular prism, construct a balanced seesaw. (As shown below)



Place two pennies on top of each other and 5 cm from the fulcrum. Place pennies at different distances from the other side of the fulcrum until the seesaw is balanced. Record your data.

Balancing the Scale

Number of Pennies	Distance from fulcrum (cm)
3	
6	
8	
10	
2	

Equation?

What type of variation exists with this data?
Explain how you know.

If you wanted the scale to be balanced with the added coins 4.5 cm from the fulcrum, how many pennies would have to be placed there?

Explain how you determined your answer.

DIRECTIONS FOR SCATTER PLOT ON TI-80

To Clear Memory:

Turn on the calculator and press **2nd, 0** to clear the memory. Press **3**[reset], **2**[reset].

To Enter Data:

1. Press the **Stat** key, then **1**[edit].
2. Enter all the x-values in the L1 column. Put one number in and press enter.
3. Press the **blue right arrow key**. Now enter the y-values under L2 in the same way the x-values were entered.

To Ready The Graph:

1. Press **2nd Stat Plots**(the y= key).
2. Press **1** to display plot1.
3. Move the cursor to highlight on and press **enter**.
4. Press the **blue down arrow key** to highlight the scatter plot,then press **enter**.
5. Press the **blue down arrow key** to highlight L1 and press **enter**.
6. Press **blue down arrow key** and arrow across to highlight L2, then press **enter**.
7. Press **blue down arrow key** to highlight the marking you want to use and press **enter**.
8. Press **window** key and make any necessary adjustments.
9. Press **graph**.

To Determine The Line Of Best Fit:

1. Press **Stat** then use the **blue right arrow key** to move cursor to [calc].
2. Press **3** to select Linreg(ax+b).
3. Press **2nd L1**, **comma** , **2nd L2**, **enter**.
4. Press **Y=** and place the cursor at y1.
5. Press **Vars**.
6. Press **2**[statistics] and arrow across 2 places to [Eq].
7. Press **5**[regeq].
8. Press **graph**.